## Mid-term Take-Home Exam

EE 508- Fall 2022 Due Wednesday November 9

## Problem 1

A notch filter based upon the Tow-Thomas Biquad is introduced in US Patent 8,436,679.
a) Obtain the transfer function for the filter in the patent.
b) Obtain an expression for the Q of this filter as a function of the Boolean input assuming there are n-bits of control
c) Design this filter for a notch frequency of 60 Hz and a bandwidth of 5 Hz
d) Comment on how this filter can be used to accurately set the notch frequency at 60 Hz .

## Problem 2

The integrator in US Patent $8,436,679$ uses an R-2R ladder instead of a single resistor resulting in a unity gain frequency that is digitally programmable. Compare the performance of this R-2R integrator with that of the basic Miller integrator.

## Problem 3

A new method of building filters is proposed in US Patent $8,952,749$ by Mediatek. Obtain the transfer function for the basic embodiment in this patent which is included in Fig. 2 and comment on the practicality of this circuit.

## Problem 4

Compare the integrator Q factor of the zero sensitivity Miller inverting integrator with that of the standard Miller Integrator. Include derivations needed to make these comparisons.


Miller Inverting Integrator


Zero Sensitivity Miller Inverting Integrator

Problem 5 A standard Miller integrator is shown below. One of the major challenges of integrating this circuit for audio frequency applications is the area required for the resistors and the capacitor. In particular, if the loss is small, the resistor $R_{\text {Loss }}$ can be very large. If a standard unit cell is used for the resistor $R$ and if a number of these are placed in series to form $R_{\text {Loss }}$, then the area for $R_{\text {Loss }}$ can be much larger than the area for $R$. And, even if there is no loss in the integrator, the area required for the resistor $R$ is often unacceptably large. For example, if in a process the sheet resistance is $R_{\square}=30 \Omega / \square$, and the resistance require to achieve a given integrator unity gain frequency is determined to be $R=30 K$, and if $R_{\text {Loss }}=20 R$, the number of squares of resistance is given by $n=n_{1}+n_{2}$ where $n_{1}$ is the number of squares for $R$ and $n_{2}$ is the number of squares for $R_{\text {Loss. }}$ In this example, $n_{1}=1000$ and $n_{2}=20,000$ so the total number of squares is $n=21,000$.


A lossy integrator with a transresistive loss element is shown below. Also shown below is a lossy integrator with two transresistive elements.
a) Derive the transfer function for the circuit with a single transresistor under the assumption $\theta R_{x} \ll R_{\text {Loss1 }}$ and show that for an appropriate $R_{\text {Loss1 }}$ and $\theta$, this lossy integrator can have the same integrator gain characteristics as the Standard Lossy Miller Integrator.
b) If $R_{x}=R / 20$ (where $R$ is the input resistance in the Standard Lossy Miller Integrator)and $\theta=0.1$, compare the total area (specified as the total number of squares )required to realize the same integrator unity gain frequency as was obtained with the Lossy Miller Integrator.
c) Determine the total area (specified as the total number of squares) required for the lossy integrator with the two transresistance elements if $R x=R / 20$ and $\theta=0.1$ if this structure has the same loss and the same integrator unity gain frequency as the Lossy Miller Integrator.
d) This process can be repeated again to further reduce the resistor area by using a two-stage transresistor ladder shown below. Is this use of transresistor elements (with one, two, or even more stages) an effective method for reducing the area requirements for building integrated audio frequency or even sub-audio frequency active filters? Discuss the benefits and/or limitations of this approach.


Lossy Miller Integrator with Transresistor Loss Element


Problem 6
Consider the following amplifier where the op amp is assumed to be ideal. Assume the amplifier is to be designed to have a gain of $16 \pm 0.5 \%$ and the layout of the resistors uses a commoncentroid geometry to cancel gradient effects. Assume the resistors are created by using a series connection of a unit resistors of 50 ohms and area $5 \mathrm{um}^{2}$. Assume the matching characteristics for closely-placed interdigitized resistors is characterized by the Pelgrom parameter $X_{R}=.01 \mu^{-1}$.
a) Determine the area for the resistors and the yield if the nominal value of $R_{2}$ is 3.2 K .
b) How does the area and yield change if the nominal value of $R_{2}$ is increased to 32 K


Problem 7 Consider the first-order RC filter shown below. Assume the resistor is comprised of a series connection of resistors with resistance density $D_{R}$ and the capacitor has a capacitance density of $D_{c}$. Assume also that $\omega_{0}$ is the $3 B$ frequency.
a) The area required to realize this filter is given by $A=A_{R}+A_{C}$ where $A_{R}$ is the area for the resistor and $A_{c}$ is the area for the capacitor. Give an expression for the area required to realize this filter in terms $\omega_{0}, D_{R}$, and $D_{C}$.
b) If the value of $R$ and $C$ are selected to minimize the total area for a given $\omega_{0}$, how does $A_{R}$ compare to $\mathrm{A}_{\mathrm{c}}$ ?


Problem 8 Consider now a requirement where two of the filters considered in the previous problem are to be designed and that the poles of these two filters are to be closely matched and are to be nominally equal to $\omega_{0}$. Assume that a common-centroid layout is used for the resistors and a common centroid layout is used for the capacitors to eliminate gradient effects. Assume the standard deviation of a resistor and of a capacitor are given respectively by the equations
$\sigma_{R}=\frac{X_{R}}{\sqrt{A_{R}}} \quad \sigma_{C}=\frac{X_{C}}{\sqrt{A_{C}}}$
where the parameters $X_{R}$ and $X_{C}$ are constants characteristic of the process and where $A_{R}$ and $A_{C}$ are the areas of the resistor and capacitor respectively. Following the notation of the previous problem, the resistance density and the capacitance density are respectively $D_{R}$ and $D_{C}$.
a) Determine the relative area of the resistor and the capacitor, $A_{R}$ and $A_{C}$ to satisfy the nominal $\omega_{0}$ requirement that will minimize the variance of the difference between the two band edges, $\omega_{01}$ and $\omega_{02}$.
b) How does the area required for minimizing variance compare to that required for minimizing area if $X_{R}=.01 u^{-1}$ and $X_{C}=.005 u^{-1}$ ?
c) How much would the variance increase if the sizing strategy to achieve minimum area were used instead of the sizing strategy to minimize variance? Assume the same values for $X_{R}$ and $X_{C}$ used in part b).


Problem 9 Design a lossy stray-insensitive noninverting switched capacitor integrator that has a nominal integrator unity gain frequency of 100 KHz , a clock frequency of 1 MHz , and a loss pole at 1 KHz where the unity gain frequency and the loss pole are specified in the z-domain. You may assume ideal switches and an ideal operational amplifier. Give the z-domain transfer function for your filter and the approximate s-domain transfer function as well.

## Problem 10 Obtain the lowest-order approximating function for a Chebyschev filter that

 meets the filter mask requirements given below. Verify your solution with a plot of the transfer function magnitude.

Problem 11 Design a $6^{\text {th }}$-order Chebyschev bandpass filter with 3 dB band edges of 1 KHz and 4 KHz and 3 dB of passband ripple. Include an expression for the transfer function of your filter and plot the filter response showing that it meets the performance requirements.

